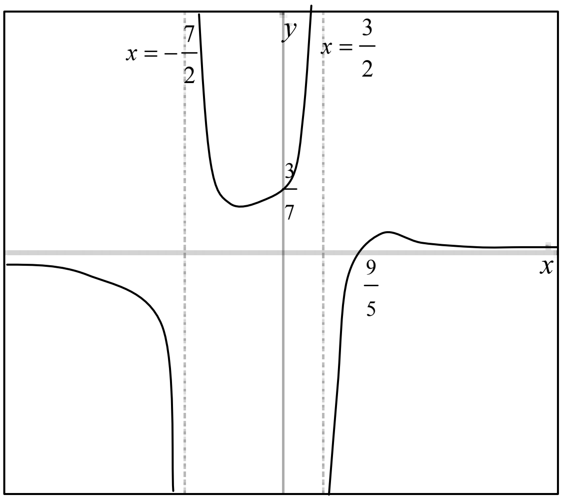
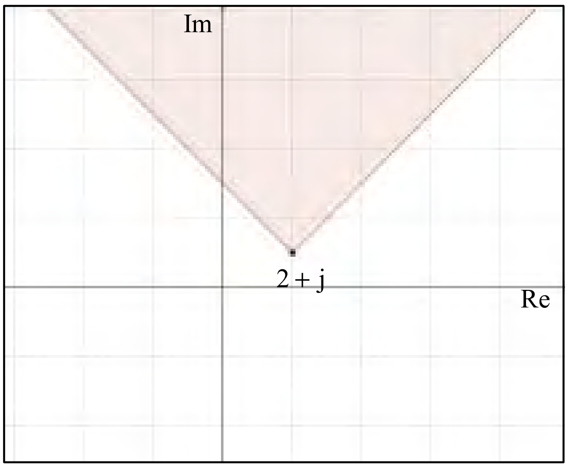
4755 (FP1) Further Concepts for Advanced Mathematics

1	$\alpha\beta = (-3+j)(5-2j) = -13+11j$	M1 A1 **[2]**	Use of $j^2 = -1$
	$\frac{\alpha}{\beta} = \frac{-3+j}{5-2j} = \frac{(-3+j)(5+2j)}{29} = \frac{-17}{29} - \frac{1}{29}j$	M1 A1 A1 **[3]**	Use of conjugate 29 in denominator All correct
2 (i)	**AB** is impossible $\mathbf{CA} = (50)$ $\mathbf{B}+\mathbf{D} = \begin{pmatrix} 3 & 1 \\ 6 & -2 \end{pmatrix}$ $\mathbf{AC} = \begin{pmatrix} 20 & 4 & 32 \\ -10 & -2 & -16 \\ 20 & 4 & 32 \end{pmatrix}$	B1 B1 B1 B2 **[5]**	 –1 each error
(ii)	$\mathbf{DB} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} -10 & -2 \\ 22 & 1 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply in correct order c.a.o.
3	$\alpha+\beta+\gamma = a-d+a+a+d = \frac{12}{4} \Rightarrow a = 1$	M1 A1	Valid attempt to use sum of roots $a = 1$, c.a.o.
	$(a-d)a(a+d) = \frac{3}{4} \Rightarrow d = \pm\frac{1}{2}$	M1	Valid attempt to use product of roots
	So the roots are $\frac{1}{2}$, 1 and $\frac{3}{2}$	A1	All three roots
	$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{k}{4} = \frac{11}{4} \Rightarrow k = 11$	M1 A1 **[6]**	Valid attempt to use $\alpha\beta+\alpha\gamma+\beta\gamma$, or to multiply out factors, or to substitute a root $k = 11$ c.a.o.

4	$\mathbf{MM}^{-1} = \frac{1}{k}\begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$	M1	Attempt to consider $\mathbf{MM}^{-1}$ or $\mathbf{M}^{-1}\mathbf{M}$ (may be implied)
	$= \frac{1}{k}\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \Rightarrow k = 5$	A1 **[2]**	c.a.o.
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}\begin{pmatrix} 9 \\ 32 \\ 81 \end{pmatrix}$	M1	Attempt to pre-multiply by $\mathbf{M}^{-1}$
		M1	Attempt to multiply matrices
	$\frac{1}{5}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}\begin{pmatrix} 9 \\ 32 \\ 81 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -10 \\ 15 \\ 85 \end{pmatrix}$	A1	Correct
	$\Rightarrow x = -2,\ y = 3,\ z = 17$	A1 **[4]**	All 3 correct s.c. B1 if matrices not used
5	$\sum_{r=1}^{n}(r+2)(r-3) = \sum_{r=1}^{n}(r^2 - r - 6)$		
	$= \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - 6n$	M1	Separate into 3 sums
	$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - 6n$	A2	–1 each error
	$= \frac{1}{6}n[(n+1)(2n+1) - 3(n+1) - 36]$	M1	Valid attempt to factorise (with n as a factor)
	$= \frac{1}{6}n(2n^2 - 38) = \frac{1}{3}n(n^2 - 19)$	A1 A1 **[6]**	Correct expression c.a.o. Complete, convincing argument
6	When $n = 1$, $\frac{n(n+1)(n+2)}{3} = 2$, so true for $n = 1$	B1	
	Assume true for $n = k$ $2 + 6 + \ldots + k(k+1) = \frac{k(k+1)(k+2)}{3}$	E1	Assume true for k
	$\Rightarrow 2 + 6 + \ldots + (k+1)(k+2)$ $= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$	M1	Add $(k+1)$th term to both sides
	$= \frac{1}{3}(k+1)(k+2)(k+3)$ $= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$	A1	c.a.o. with correct simplification
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$ it is true for $n = k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $n = 1$, it is true for $n = 1, 2, 3$ and so true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1

7 (i)	$x = \frac{-7}{2},\ x = \frac{3}{2},\ y = 0$	B1 B1 B1 [3]	
(ii)	Large positive x, $y \to 0^+$ (e.g. consider $x = 100$) Large negative x, $y \to 0^-$ (e.g. consider $x = -100$)	B1 B1 M1 [3]	Evidence of method
(iii)		B1 B1 B1 [3]	Intercepts correct and labelled LH and central branches correct RH branch correct, with clear maximum
(iv)	$x < -\frac{7}{2}$ or $\frac{3}{2} < x \le \frac{9}{5}$	B1 B2 [3]	 Award B1 if only error relates to inclusive/exclusive inequalities

8(a) (i)	$\lvert z-(2+6j)\rvert=4$	B1 B1 B1 [3]	2 + 6j seen (expression in z) = 4 Correct equation
(ii)	$\lvert z-(2+6j)\rvert<4$ and $\lvert z-(3+7j)\rvert>1$	B1 B1 B1 [3]	$\lvert z-(2+6j)\rvert<4$ $\lvert z-(3+7j)\rvert>1$ (allow errors in inequality signs) Both inequalities correct
(b)(i)		B1 B1 B1 [3]	Any straight line through 2 + j Both correct half lines Region between their two half lines indicated
(ii)	$43+47j-(2+j)=41+46j$ $\arg(41+46j)=\arctan\left(\frac{46}{41}\right)=0.843$ $\frac{\pi}{4}<0.843<\frac{3\pi}{4}$ so $43+47j$ does fall within the region	M1 A1 E1 [3]	Attempt to calculate argument, or other valid method such as comparison with $y=x-1$ Correct Justified

9 (i)	$\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}$ $=\frac{2(r+1)(r+2)-3r(r+2)+r(r+1)}{r(r+1)(r+2)}$ $=\frac{2r^2+6r+4-3r^2-6r+r^2+r}{r(r+1)(r+2)}=\frac{4+r}{r(r+1)(r+2)}$	M1 A1 **[2]**	Attempt a common denominator Convincingly shown
(ii)	$\sum_{r=1}^{n}\frac{4+r}{r(r+1)(r+2)}=\sum_{r=1}^{n}\left[\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}\right]$ $=\left(\frac{2}{1}-\frac{3}{2}+\frac{1}{3}\right)+\left(\frac{2}{2}-\frac{3}{3}+\frac{1}{4}\right)+\left(\frac{2}{3}-\frac{3}{4}+\frac{1}{5}\right)+..$ $..+\left(\frac{2}{n-1}-\frac{3}{n}+\frac{1}{n+1}\right)+\left(\frac{2}{n}-\frac{3}{n+1}+\frac{1}{n+2}\right)$ $=\frac{2}{1}-\frac{3}{2}+\frac{2}{2}+\frac{1}{n+1}-\frac{3}{n+1}+\frac{1}{n+2}$ $=\frac{3}{2}-\frac{2}{n+1}+\frac{1}{n+2}$ as required	M1 M1 A2 M1 A1 **[6]**	Use of the given result (may be implied) Terms in full (at least first and one other) At least 3 consecutive terms correct, -1 each error Attempt to cancel, including algebraic terms Convincingly shown
(iii)	$\frac{3}{2}$	B1 **[1]**	
(iv)	$\sum_{r=50}^{100}\frac{4+r}{r(r+1)(r+2)}$ $=\sum_{r=1}^{100}\frac{4+r}{r(r+1)(r+2)}-\sum_{r=1}^{49}\frac{4+r}{r(r+1)(r+2)}$ $=\left(\frac{3}{2}-\frac{2}{101}+\frac{1}{102}\right)-\left(\frac{3}{2}-\frac{2}{50}+\frac{1}{51}\right)$ $=0.0104$ (3s.f.)	M1 M1 A1 **[3]**	Splitting into two parts Use of result from (ii) c.a.o.